Lesson plan format

1. Title of research lesson: Non-linear Tile Exploration

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3. Mathematical content/concepts addressed

Foundation: Knowledge of area, patterning skills including linear, and basic algebraic skills including writing algebraic expressions

Focus: Non-linear patterns

Future: Quadratic Functions

4. Objectives:

Content goals of the lesson

- Discover non-linear patterns and express them in multiple ways
- Introduced to non-linear functions

Common Core Standards for Mathematics

- *Algebra: Seeing Structure in Expressions A-SSE*

Interpret the instruction of expressions

1. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

- *Algebra: Creating Expressions A-CED*

Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- *Algebra: Building Functions F-BF*

Build a function that models a relationship between two quantities.

1.a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Broader content goals of the unit

- Write variable expressions to represent patterns
- Identify quadratic patterns in areas and extend them to factoring
- Recognize equivalency of quadratics in various forms including standard form and factored form

Goals for mathematical practice

Common Core Standards: Mathematical Practice

- Standard 1: Make sense of problems and persevere in solving them.

Students will be working on a tile problem and sharing their results in multiple ways.

- Standard 2: Reason abstractly and quantitatively.

Students may extend a pattern to write a rule.

- Standard 3: Construct viable arguments and critique the reasoning of others.

Students will explain and justify their representations of the patterns both orally and visually.

- Standard 4: Model with mathematics.

Students will extend the pattern using representations which may include tiles, tile drawings, tables, and rules.

Long-term student development goals

- Students will persevere when faced with a mathematical problem.
- Students will work collaboratively and cooperatively.
- Students will communicate their reasoning both verbally and in written form.

5. Research base:

Algebra Connections, College Preparatory Mathematics, CPM Educational Program, Volume 1, Dietiker, Kysh, Sallee, Hoey, 2006.

Common Core State Standards for Mathematics, July 2010.

6. Materials needed:

*Overhead of linear tile pattern
*Resource page of enlarged tile patterns
*Tile pattern Worksheet
*Tiles
*Document Camera
*Colored pencils

7. Instructional sequence

Learning activities & Key questions (with timing)	Expected student responses, questions, misconceptions	Teacher's support (follow-up questions/actions)	Formative assessments
Welcome (2 min) (Set up with materials including the enlarged Tile Pattern Team Challenge resource page, tiles, and colored pencils)		Welcome students and introduce yourself. Explain that the Lesson Study participants are here to observe students' mathematical thinking.	
Task 1 (5 min) Dave has been working with tile patterns and came up with the following tile pattern. (Start out with a linear tile pattern on the overhead showing figures 1,2, 3.)			
How is this pattern growing?	Students may not recognize the growth.	How many more tiles have been added to create the next figure?	Students should state the growth (number of tiles).
What would the figure before figure 1 look like?	Students may not think that there is a figure before figure one or they may not know how to generate it.	If you added tiles to create the next figure, what would you do to find the previous figure?	Students should describe Figure 1 in terms of shape or by the growth.
What could we call it? (Figure 0)	Students may not realize to call it figure 0 and just think of it as the previous figure.	If the figure after this one is figure 1, what could this figure be called?	Students should refer this figure as figure 0.

Task 2 (25 min) Let's look at some other tile patterns. (Hand out the enlarged Tile Pattern resource page *1 per team*).			
You have 30 seconds to study the pattern and then share with your group how you see the pattern is growing. (Hand out Tile Pattern Resource page).	Students may not be able to see the pattern.	Where does the figure appear to be changing? How has it changed between figure 1 and figure 2? How many more tiles have been added to create the next figure?	Students should state the growth (number of tiles) and may state how and where the tiles are being added.
Now, create figure 0 and figure 4 on your paper or with the tiles.	Students may have difficulties working backwards to figure 0.	If you added tiles to create the next figure, what would you do to find the previous figure?	Students should generate figures 0 and 4.
Record the number of tiles in figures 0-4 and compare with your group.	Students may miscount or have an incorrect shape.	Does everyone in your group get the same number of tiles for figure #?	Students should record the number of tiles for each figure number.
How many tiles are in figure 100? Show your results by sketching figure 100 and show support for your findings.	Students may just find the answer and not justify it with more than one representation or any at all. Students may not persevere if they begin to struggle.	What does it look like? How many tiles will it have? Encourage students to try each representation.	Students should show the number of tiles in figure 100 as well as multiple representations to justify their results.
Task 3 (15 min) Write an expression for the number of tiles in figure n: n being any figure number	Students may not be able to generalize a rule using a variable.	Can you break the shape apart into separate pieces? How can the graph, table or pattern help?	Students should attempt to create a rule.
Verify that your expression works for figure 3.	Students may not know where to put 3 into their expression to see how many tiles should be in Figure 3.	How can you show your expression works? Will it work for Figure 3?	Students should verify that Figure 3 has the number of tiles they stated it did in Task 2.
Task 4 (10 min) It's time to share our findings. Everyone please come up when your team presents. (Choose groups to present their Figure 100 figure and then choose different groups to present their expressions).	Students may not be comfortable presenting or confident in their answers or explanations.	Validate the student responses and ask questions to get complete explanations.	Students should justify and explain the representations they used to describe figure 100 and the process used to create their rule.

Closure (3 min)		
Describe how your pattern and the other patterns presented differ from the Dave's pattern shown at the beginning?	Students may not make the connection. Students may ask about other non-linear patterns.	All students should recognize that the tile patterns represent non-linear functions.

8. Additional information:

Source: Matthews, M.E., Hlas, C.S., & Finken, T.M. (2009). Using Lesson Study and four-column lesson planning with preservice teachers. *Mathematics Teacher*, *102*, 504-508.