

1. Title of research lesson: Less is More – The Meaning of Minus

2. Authors: Colleen Chmelik, Phil Graser, Randy Gustafson, Jessica Kelly, Nancy Schlageter

3. Mathematical content/concepts addressed –

*Foundation:* Understanding of basic subtraction, positive and negative numbers, opposites

*Focus:* Modeling subtraction with negative numbers and gathering information about student’s intuitive sense of subtraction.

*Future:* This lesson provides the foundation for understanding integer operations.

4. Objectives: (Content Goals of the Lesson)

Explore and develop strategies to subtract integers.

The purpose of this less is research based, rather than a lesson geared toward teaching a particular concept. We will gather information about how students view subtraction and their ability to apply various strategies. This was pulled from CCS for Practice #4 as well as observations based on our student probe.

5. Research base:

CPM – Making Connections

CMP – Accentuate the Negative

Lorna Vazquez, Chris Hlas, Kate Masarik, Michelle Parks

6. Materials needed:

White board – sections for different strategies.

Laminated Strategy Labels

Laminated + / -

Handouts (3-8 = -5 and exit slip)

Tiles (for students)

Observation sheets

+ /- tiles

## 7. Instructional sequence

Learning activities & Key questions (with timing)	Expected student responses, questions, misconceptions	Teacher's support (follow-up questions/actions)	Formative assessments
<p>What does minus mean? As in the problem <math>8 - 3</math>.            What type of situation would be a "minus" situation?</p> <p>Does subtracting always results in a smaller value (number)? What if I subtract what someone owes to me? Do they have less money?</p>	<p>Take away            Difference            Remove            Subtract</p>	<p>Give them time to write a few words down before prompting for answers.</p> <p>Place a web diagram on board and place various strategies around the web. Leave this for students to reference.</p>	<p>Write down on your paper the word that comes to mind.</p>
<p>Before you turn your papers over – does anyone know the value of <math>3-8</math>? Most of you have already worked with negative numbers. Can you explain or show that <math>3-8 = -5</math>? As many ways as you can.            [Handout]</p>	<ul style="list-style-type: none"> <li>• Number line &amp; counting backwards</li> <li>• <math>3-3 = 0</math> <math>0-5 = -5</math> (bench marks)</li> <li>• Different representations of the problem: <math>2-7 = -5</math>, <math>1-6</math>, <math>4-9</math>, <math>5-10</math> (Relationship)</li> <li>• Make up a story problem or situation</li> <li>• Find a pattern</li> <li>• Fact families</li> <li>• Switch the numbers <math>8-3 = 5</math> (Distance)</li> <li>• Comparisons (height) (Difference)</li> </ul>	<p>Teacher - Take notes during work time to scaffold results. Look for levels of complexity. Have students write their strategies in specified areas on white board or post-it paper while others are working.</p>	<p>Write down on your papers as many ways as you can think of to represent that.</p>
<p>Have students share their responses – scaffold by level of complexity/sophistication.</p> <p>Pull all these strategies together and tell them that today we are going to show them another model. Sometimes certain models work better than others. Example: A physical model like apples doesn't work when you take 5 apples away from 3 apples.</p>	<p>Same as above</p>	<p>Ask clarifying questions during strategy presentation.</p> <p>Label the different strategies and leave them posted on board.</p> <ul style="list-style-type: none"> <li>• Number line</li> <li>• Story Problem</li> <li>• Pattern</li> <li>• Fact Family</li> <li>• Difference</li> <li>• Take Away / Tiles</li> <li>• Distance</li> <li>• Bench Marks</li> </ul>	<p>If students are writing down other strategies, have them draw a line under their initial responses.</p>

Introduce additional strategies not modeled by the students:

1. Story/situation problem
2. +/- tiles (zero pairs)
3. Number sentence
4. Number line or counting backwards.

Make a point to bring in another word problem to discuss with the class.

Example: James Starcks gained 3 yards on first down. On second down he lost 8 yards. How many yards does he have now?

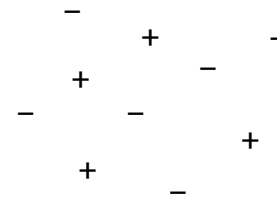
Present +/- and zero pair.

- Here is another mathematical model to represent integers. Do you know what the word “integer” means? (Whole numbers and their opposites)
- Show them the +, what do you think this represents?
- Show them the -, what do you think this represents?
- Show them the - and + together, what does this represent?
- Draw the model on the board. “what is the value of this combination?”
- On the back of your paper draw another model to represent -2.
- Try another way. (Have a few kids demonstrate their arrangement).
- Use an example from the board with three or more + pieces.
- "What happens if three + pieces are removed? “ (you have -5)
- “What if we started with -, - ? How would we remove three + pieces?”
- How can we model  $-2 - 6$ ?
- Show them how to add the zero pair so that you have 6 +'s to take away.

Hand out the tiles. Have students work in groups of 3-4. Explain that the +'s mean “1” and the -'s mean -1.

Students may not realize the value is the answer to the expression. Be sure to write the expression that accompanies the model so student connect the two parts.

Add a model of this strategy to the models the students present.



Note how students strategize on how to model the  $-2-6$  problem.

<p>Model with tiles.  -2-4  0 - 4  -4 - 2  -4 - 2  -2 - (-5)</p>	<p>Now let's try a few on our own.  Students work individually for 5 min  In pairs, discuss response.</p> <p>"Write down your models on the back of your piece of paper so that you can show me what you did as I walk around."</p> <p>Put the first three problems on the board.</p>	<p>Have students write down their models on the back of the 3-8 problem. Fold paper in fourths and use the boxes to draw the models.</p>	<p>Teacher observes students working. Offers support as needed. Clarifies misconceptions.</p>
<p>Give students a card with a representation of a "subtraction" problem. Thinking of the strategies that we've discussed, can you show three different ways to represent the expression on your card?</p>	<p>Work individually and then pair.</p>	<p>Card says - 7 - 3.</p>	<p>Exit slip. Evaluate the slips to see if students changed <b>or added to</b> their initial strategies for modeling.</p>
<p>Closing/Summarize</p>	<p>Thank you for being here today. I hope you will leave thinking about the different ways we discussed to model subtraction problems and if you ever forget some of the subtraction rules you've learned, you can use the models to help make sense of the situation.</p>		

8. Additional information/ Standards Connection:

- Standards Connection:
  - Standards for Mathematical Practice: 2, 3, 4
  - 7.NS 1a-d
- Relevant mathematical vocabulary: integers, opposites, zero, subtract, mathematical, model, represent, number line, difference

9. Content Goals:

- Explore & develop strategies to subtract integers

10. Broader content goals of the unit (Pre/Post Knowledge):

- Relationship of adding & subtracting
- Property of Subtraction
- Understand same concepts work for all #'s

- Flexibility in representations

#### Goals for Mathematical Practice

- Model with mathematics
- Construct and present viable arguments and critique the reasoning of others.
- Develop flexibility in utilizing a variety of mathematical models and methods.

#### Long term student development goals:

- Flexibility in strategies
- Engage in problem
- Discuss and share
- Open to different methods of solving

**Observation Forms:**

Activity 1: What is minus?

a. Observe the number of kids who respond with various thinking (one word for every kid) – Tally responses

Taking things away	Subtracting	Difference between	Range	How much less	Opposite	Opposite of addition	Other (be specific)

Activity 2: For each student record first strategy used, then keep track of others. Check sheet or list with numbers.

1. Number line & counting backwards
2.  $3-3 = 0$   $0-5 = -5$  (bench marks)
3. Different representations of the problem:  $2-7 = -5$ ,  $1-6$ ,  $4-9$ ,  $5-10$  (Relationship)
4. Make up a story problem or situation
5. Find a pattern
6. Fact families
7. Switch the numbers  $8-3 = 5$  (Distance)
8. Comparisons (height) (Difference)

Student	1 <sup>st</sup> strategy	2 <sup>nd</sup> strategy	3 <sup>rd</sup> strategy	4 <sup>th</sup> strategy	5 <sup>th</sup> strategy

### Activity 3: Presentation of strategies:

- Word problem – observe their thinking/strategies as they work through.
- Demonstration of the three modeling problems (Comments section – open ended)
  - a. Set up the tiles correctly
  - b. Do they understand the “zero”?

Activity 4:  $-2 - 6$  (or other practice problem)  
Note student thinking during this activity.

Activity 5: Model the practice problems of 0-4, etc.



Activity 6: Exit slip with different representations

- b. Note ability to model the situation with various representations.
- c. Note student name so we can compare to first strategy (activity 1)

Student ID	Observation of process <ul style="list-style-type: none"><li>• First strategy used</li><li>• Willingness to use additional strategies</li><li>• Fluency of strategies</li></ul> Note when students pair <ul style="list-style-type: none"><li>• Key Vocabulary</li><li>• Explanation of shared strategies</li></ul>

To Do:

Phil: Get tiles (400)

Put them in baggies

Colleen: Make labels and laminate and put magnets on them

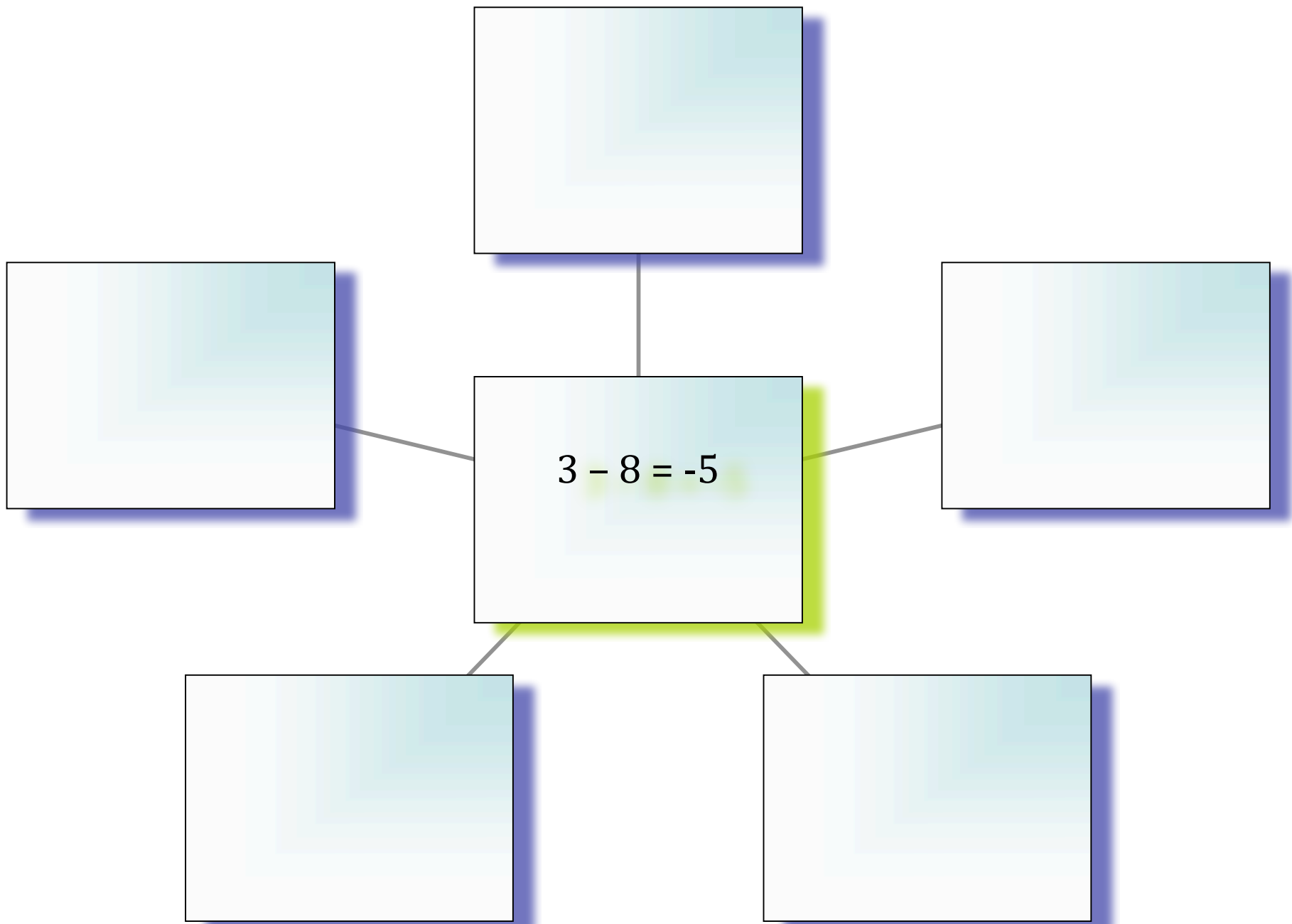
Big pluses and minus' with magnets (30 each)

Jessi: Print off hand out for  $3-8 = -5$  (web diagram)

Print off exit slip

Colleen: Copies of the observation forms with clipboards

Source: Matthews, M.E., Hlas, C.S., & Finken, T.M. (2009). Using Lesson Study and four-column lesson planning with preservice teachers. *Mathematics Teacher, 102*, 504-508.



Story Problem

Number line

Pattern

Fact Family

Difference

# Take Away/Tiles

## Distance

### Bench Marks